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Sticking between Fine Particles by Capillary Bridges

液架橋による粒子間固着作用

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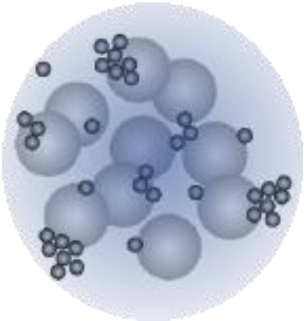
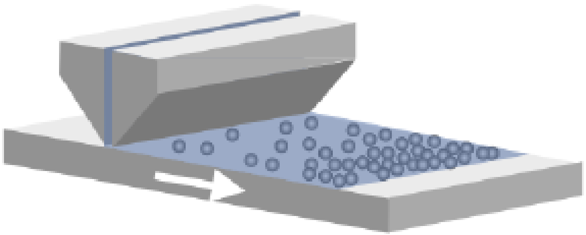
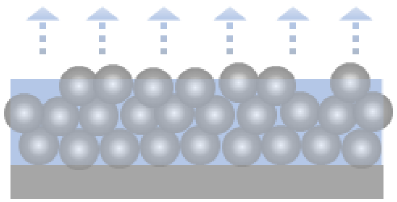
Material Fabrication from Colloidal Suspensions



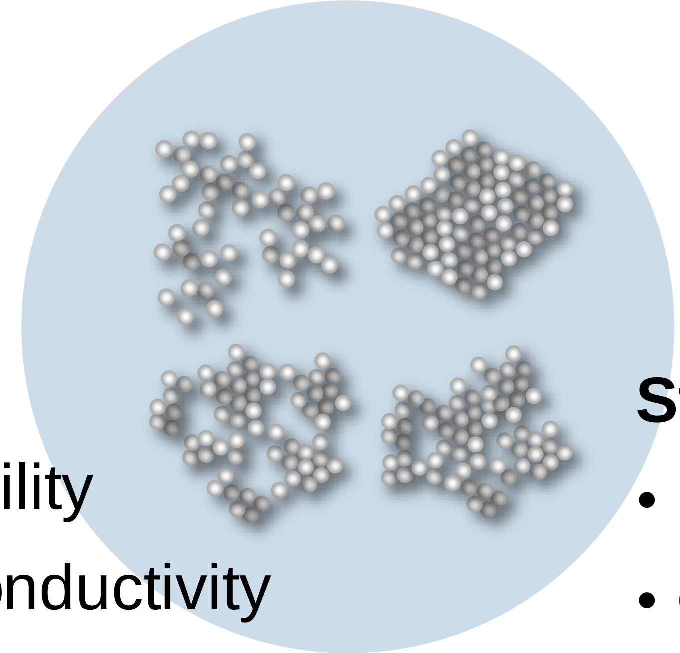
Dispersing



Coating



Drying



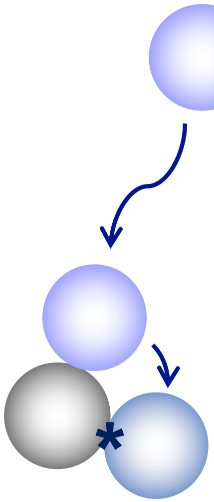
Functions

- Strength • Permeability
- Electrical/Thermal conductivity
- Optical property

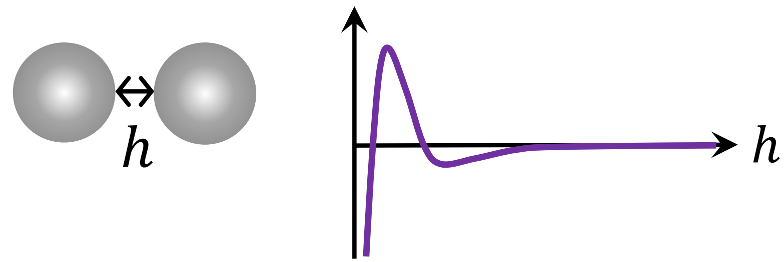
Structures

- Porosity
- Contact network

Aggregate Formation

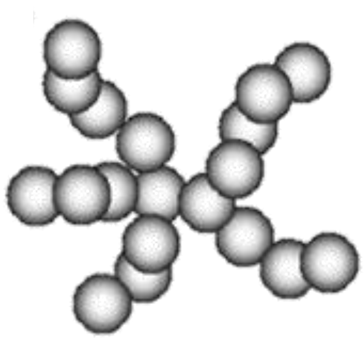


1. Collision (Diffusion) ← Interparticle potential

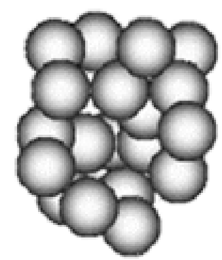


2. Sticking (Reaction): Fixation of contact points

← Interparticle slip/rolling friction

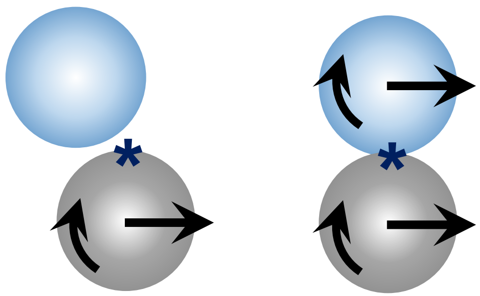


Diffusion-limited

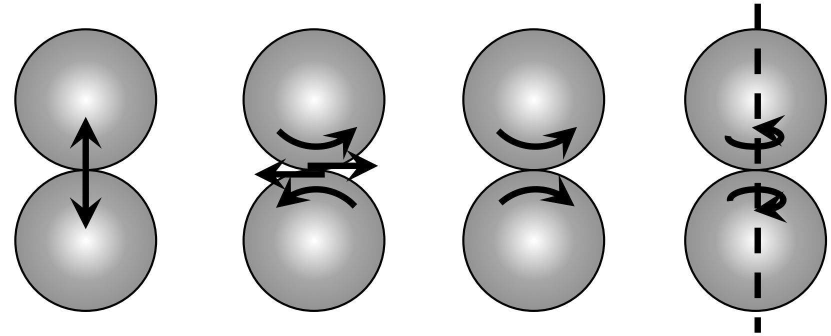


Reaction-limited

Modeling of Sticking



Slip **Stick**



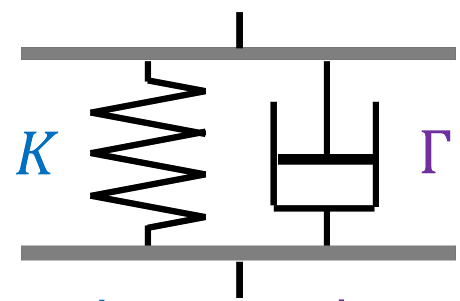
Normal Shear Bending Twist

Constraint on the relative motions

Contact force/torque

$$F = K\delta + \Gamma\dot{\delta}$$

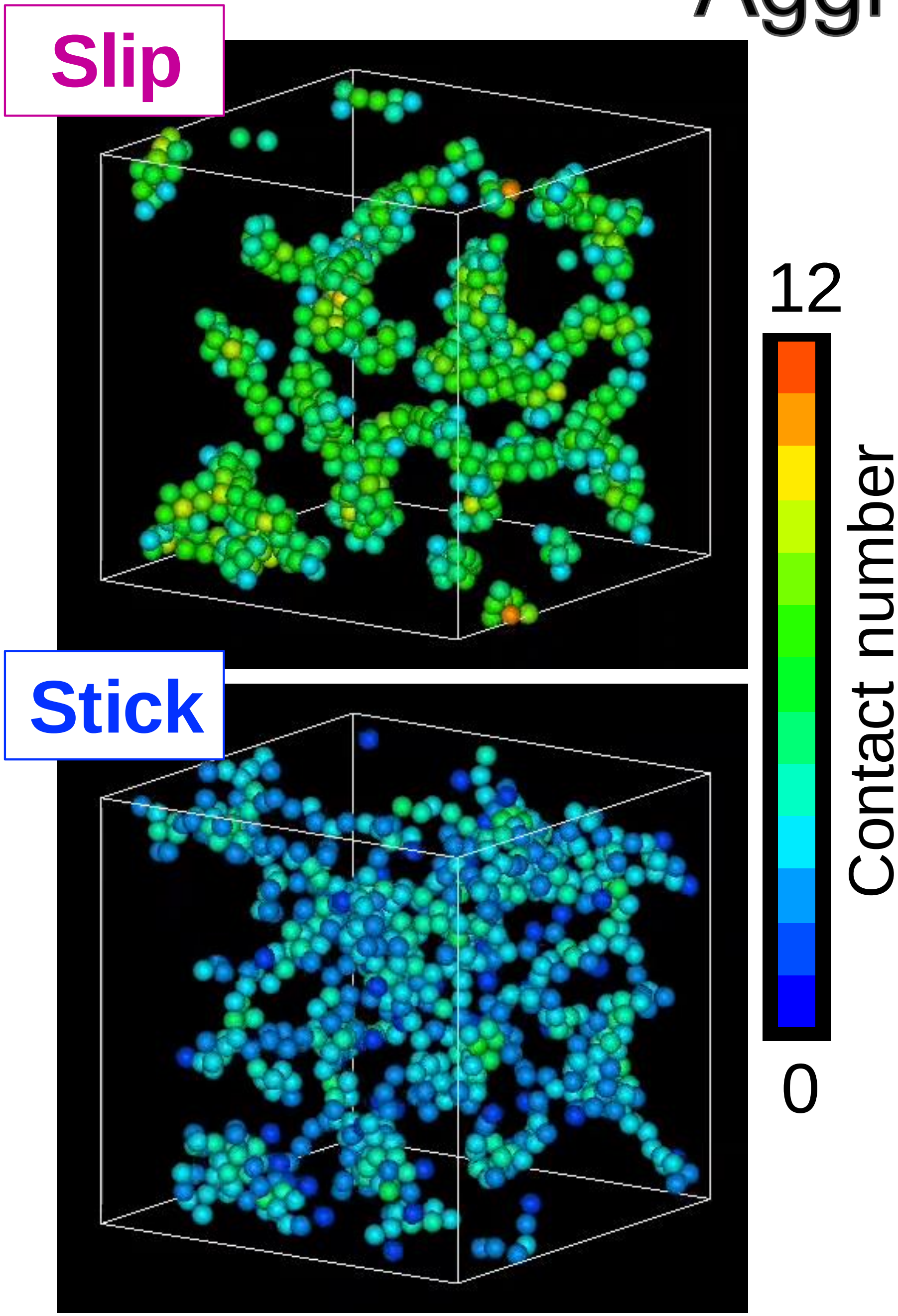
δ: Displacement

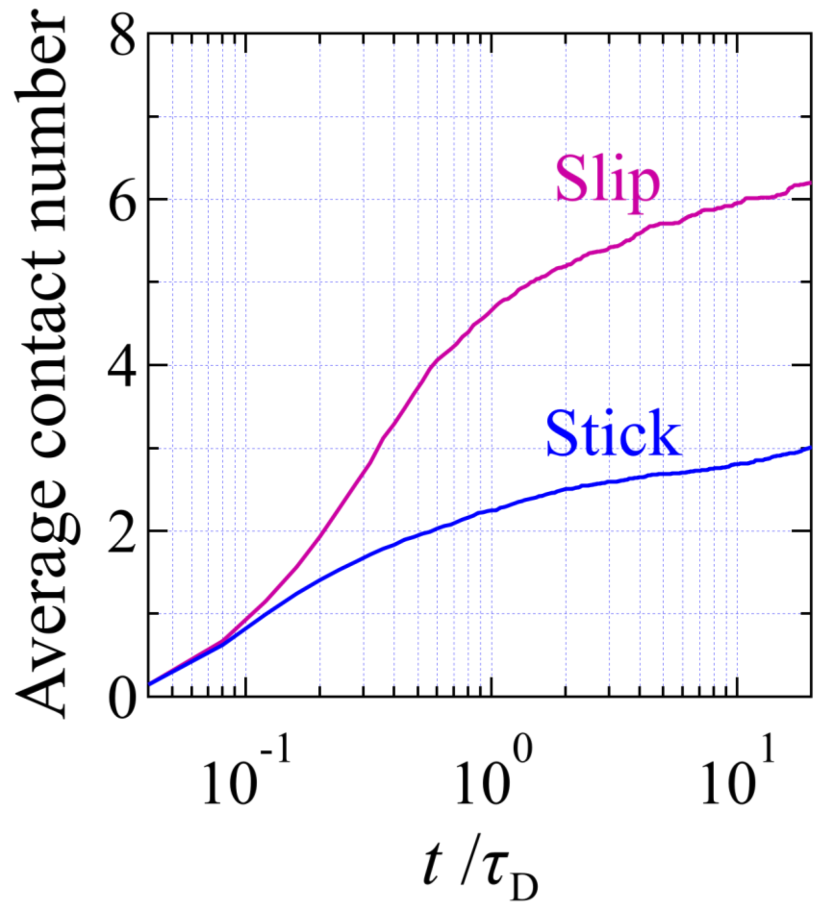


Elastic spring Viscous damper

Aggregation

Langevin eq.



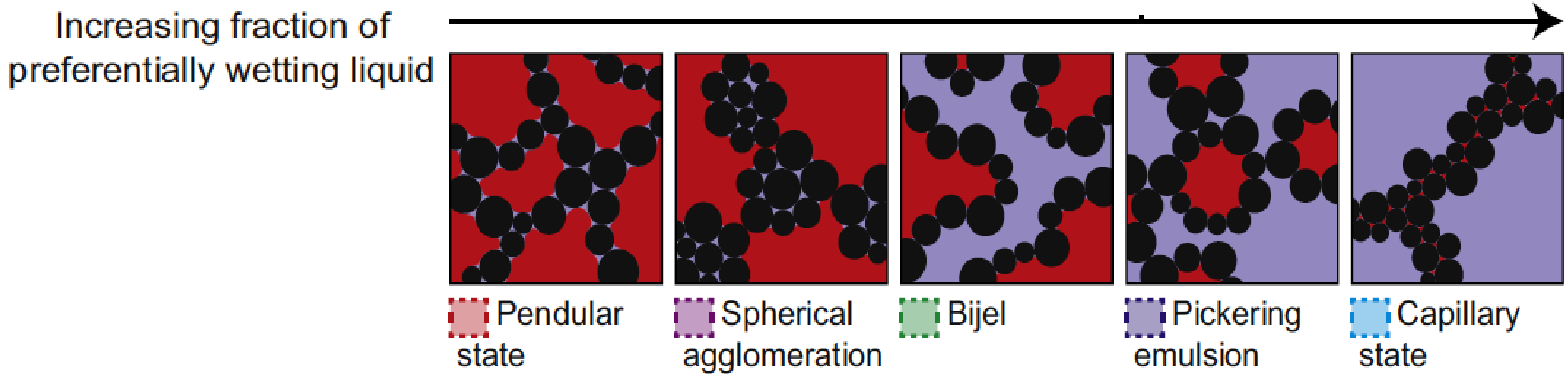


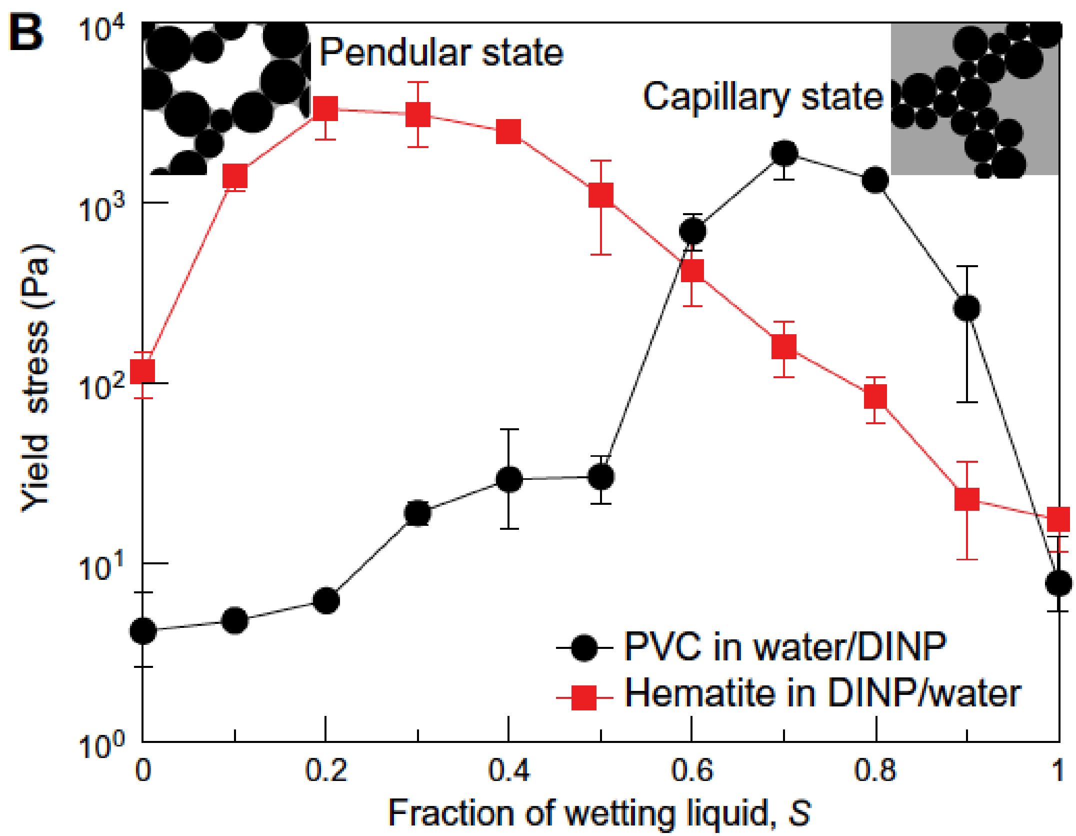
Timescale of particle diffusion:

$$\tau_D = d^2/D$$

Particle diameter: d Diffusion coefficient: D

A Possible Origin of Sticking

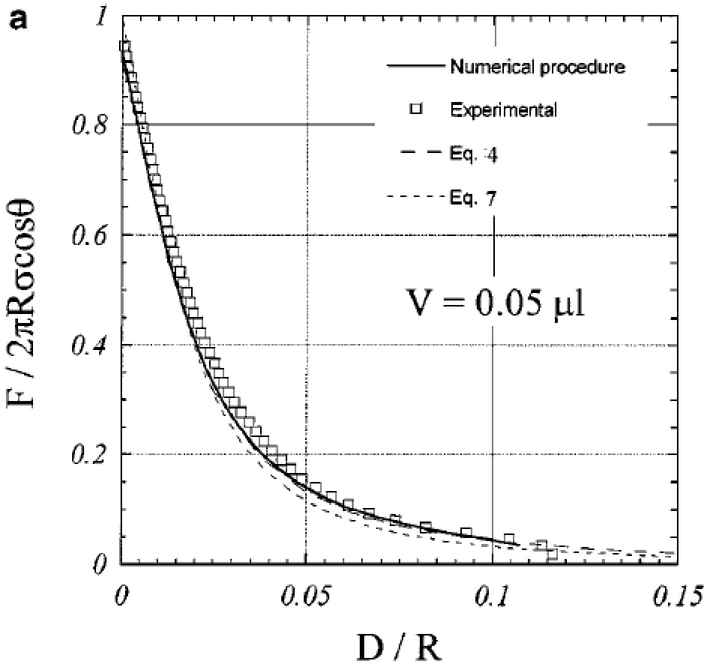


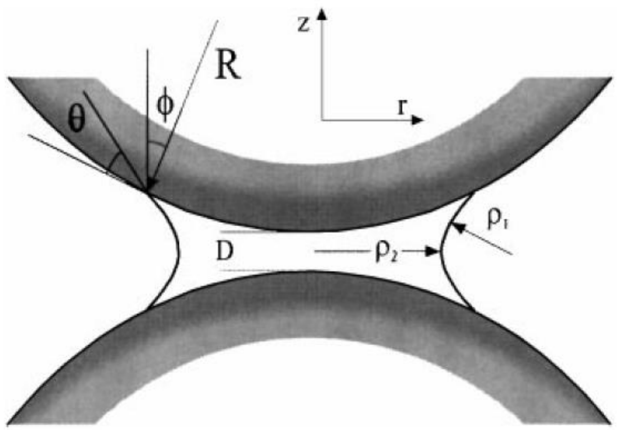


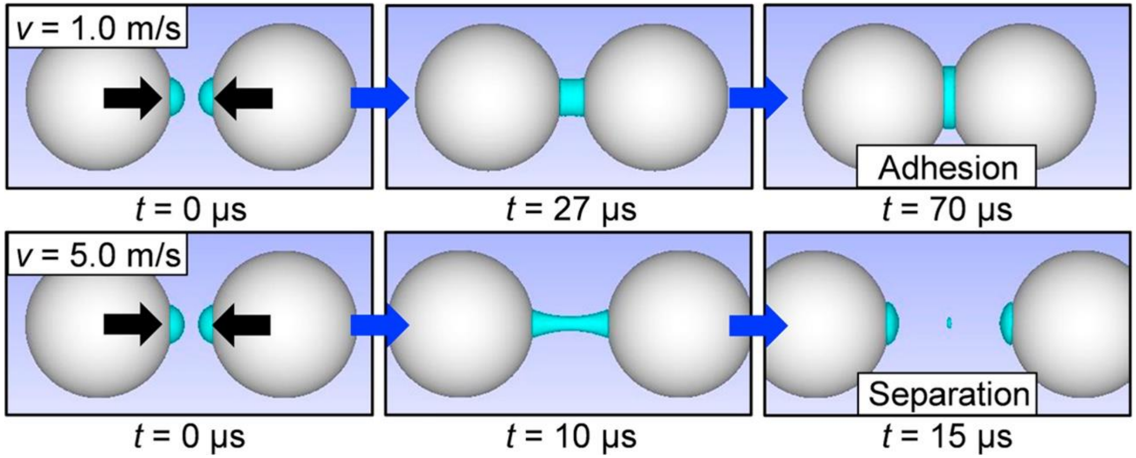
Network formation by capillary bridges

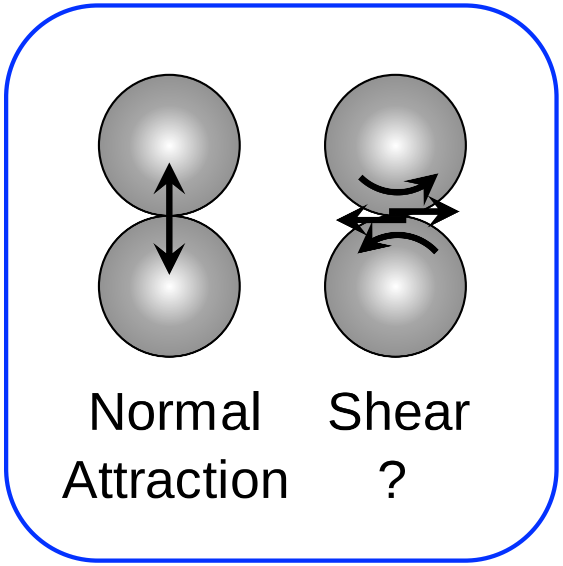
E. Koos,
Curr. Opin. Colloid Interface Sci.
19 (2014).

Capillary Bridge Force





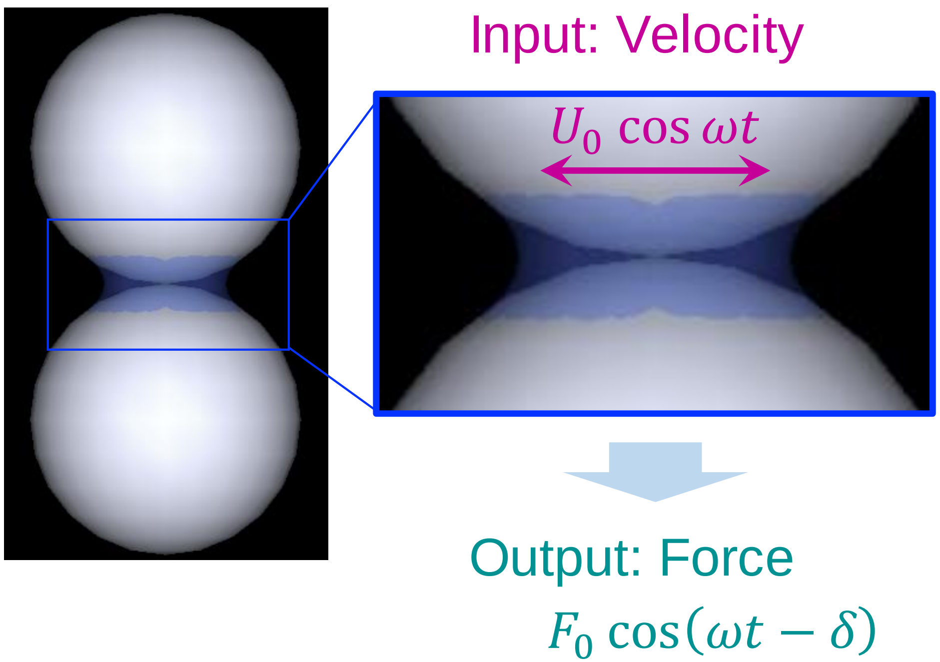


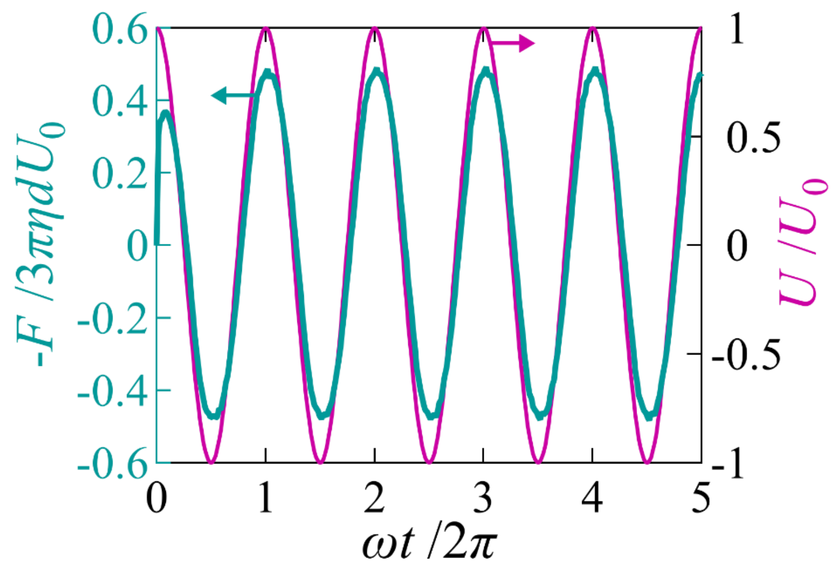


O. Pitois, P. Moucheront, X. Chateau, J. Colloid Interface Sci. **231** (2000).
H. Kan, H. Nakamura, S. Watano, Chem. Eng. Sci. **138** (2015).

Objective

- Examination of sticking by capillary bridges
- Calculation of the viscoelasticity of shear force induced by capillary bridges





Equations

Fluid flow $\nabla \cdot \boldsymbol{v} = 0$

$$\rho\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v}\cdot\nabla\boldsymbol{v}\right) = \nabla\cdot\boldsymbol{\sigma} + \Phi\boldsymbol{K}$$

Imposition of velocity in particle domains: $\Phi\boldsymbol{v} = \Phi\boldsymbol{V}$

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + [\nabla\boldsymbol{v} + (\nabla\boldsymbol{v})^{\mathrm{T}}] + \gamma(\boldsymbol{I} - \hat{\boldsymbol{n}}\hat{\boldsymbol{n}})|\nabla f|$$

Surface tension

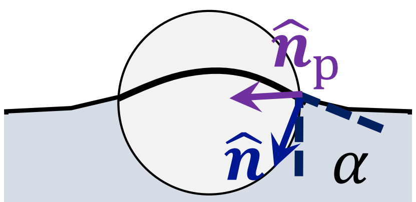
Free surface

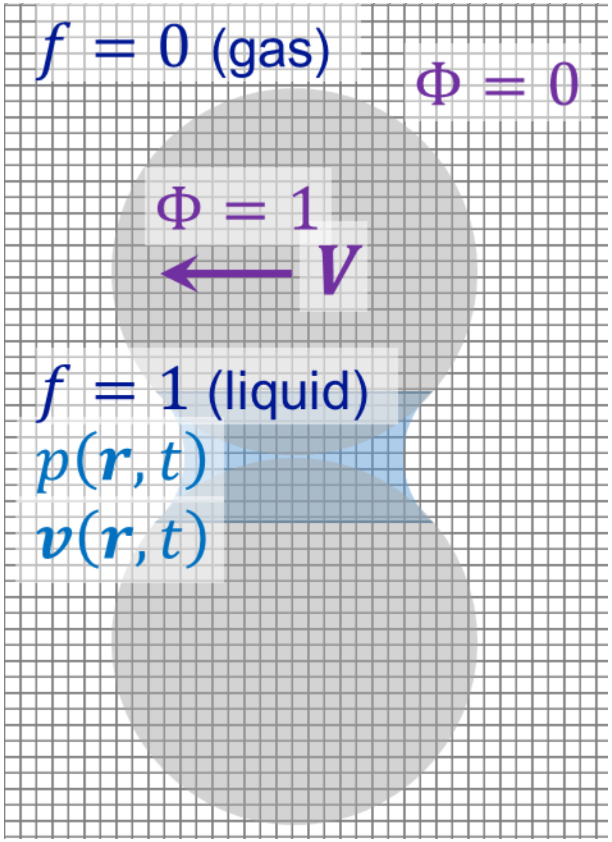
$$\frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\nabla f = 0$$

Contact angle condition

$$\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{n}}_{\mathrm{p}} = \cos\alpha$$

$$\hat{\boldsymbol{n}} = \nabla f/|\nabla f|$$





Drag force

$$\boldsymbol{F} = \int \Phi\nabla\cdot\boldsymbol{\sigma}\,\mathrm{d}\boldsymbol{r}$$

M. Fujita, O. Koike, Y. Yamaguchi, J. Comput. Phys. **281** (2015).

Simulation Conditions

Particles

- Diameter: $d = 100\ \mathrm{nm}$
- Contact angle: $0°$

Fluid: water

- Volume: $0.012d^3$
- Surface tension: $\gamma^* = 0.18 - 7.2$

$$\gamma^* = \frac{\rho d}{\eta^2}\gamma$$

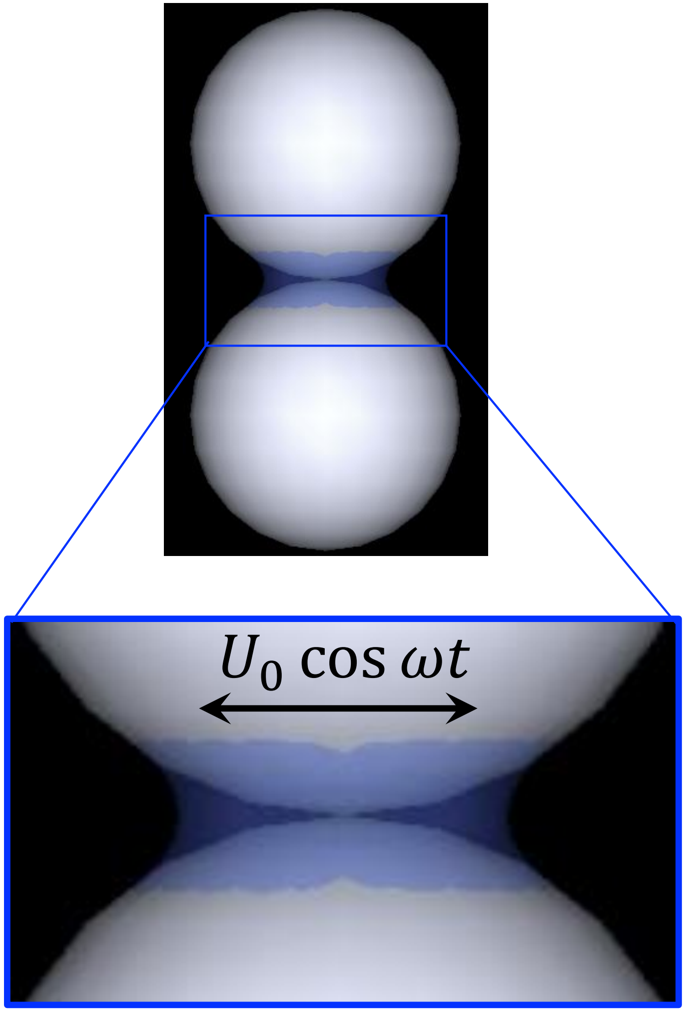
Fluid density: ρ
Viscosity: η

Motion

- Displacement: $\delta_0 = 0.01d$
- Velocity amplitude: $U_0 = \delta_0 \omega$
- Frequency: $\omega\tau_{\mathrm{vis}} = 3 - 300$

$$\tau_{\mathrm{vis}} = \frac{\rho d^2}{\eta}$$

Timescale of momentum diffusion by viscosity



Dynamic Drag Coefficients

Input: Velocity

$$U(t) = U_0 \cos \omega t$$

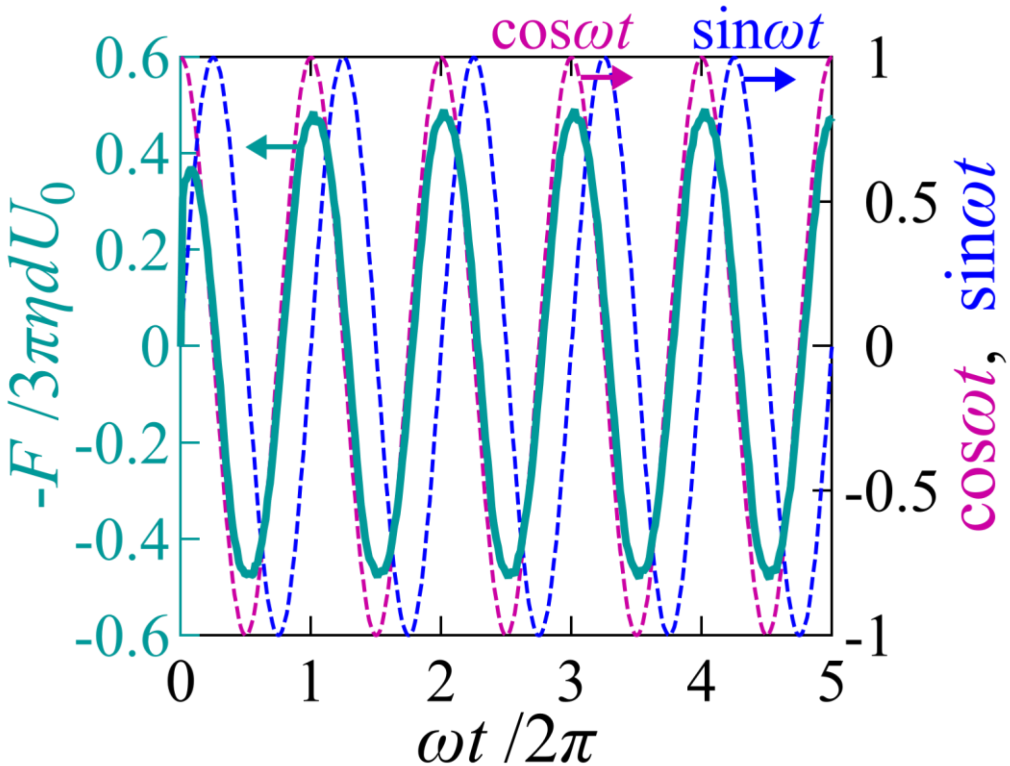
Output: Force

$$F(t) = F_0 \cos(\omega t - \delta)$$

$$= -U_0(\zeta' \cos \omega t + \zeta'' \sin \omega t)$$

$$\zeta' = \frac{F_0}{U_0} \sin \delta \qquad \zeta'' = \frac{F_0}{U_0} \cos \delta$$

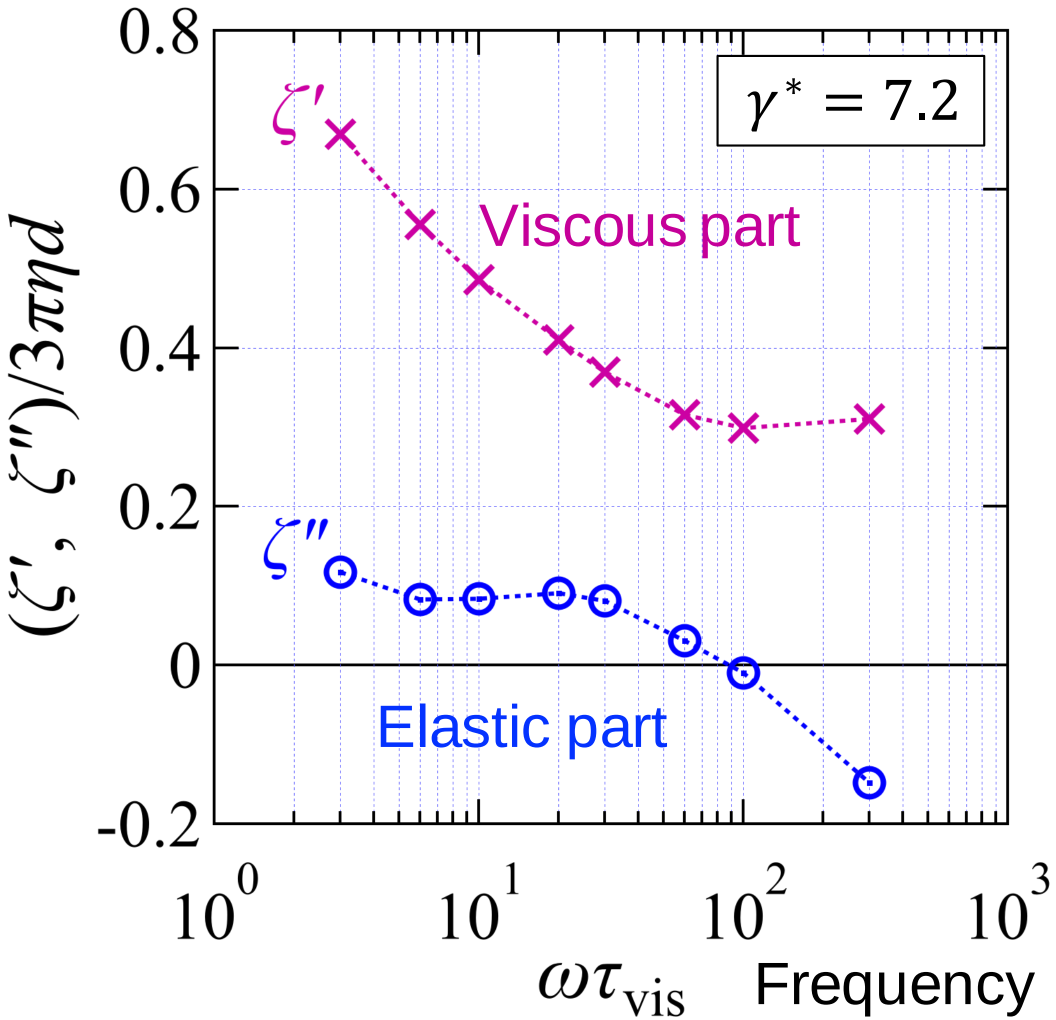
Viscous part — Elastic part



Calculation of dynamic drag coefficients

$$\zeta'(\omega) = -\frac{\omega}{\pi U_0} \int_0^{2\pi/\omega} F(t) \cos \omega t \, \mathrm{d}t \qquad \zeta''(\omega) = -\frac{\omega}{\pi U_0} \int_0^{2\pi/\omega} F(t) \sin \omega t \, \mathrm{d}t$$

Dynamic Drag Coefficients

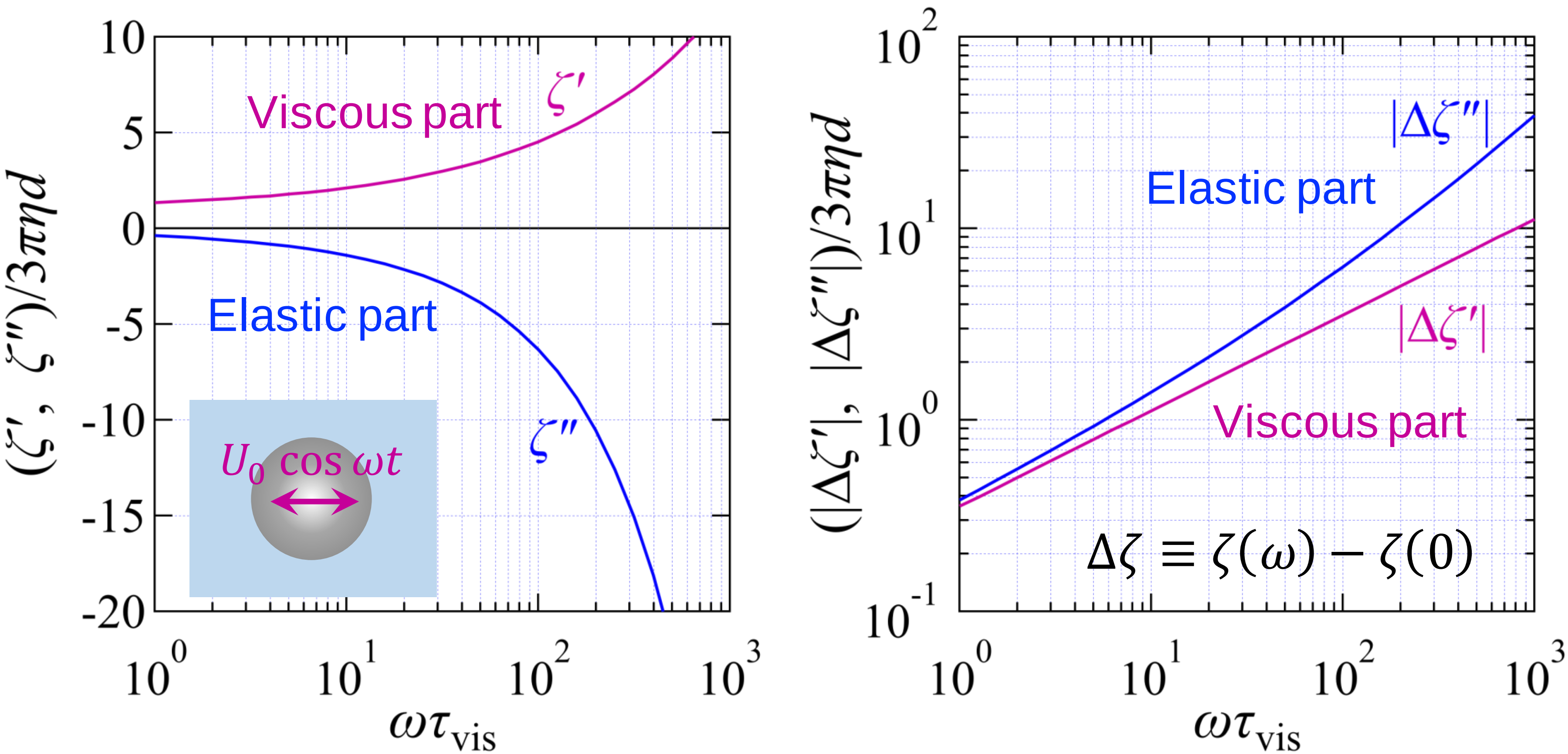


- Viscoelasticity of shear force by capillary bridges
- Negative elastic part at high frequency

Drag Coefficients in Bulk Fluid

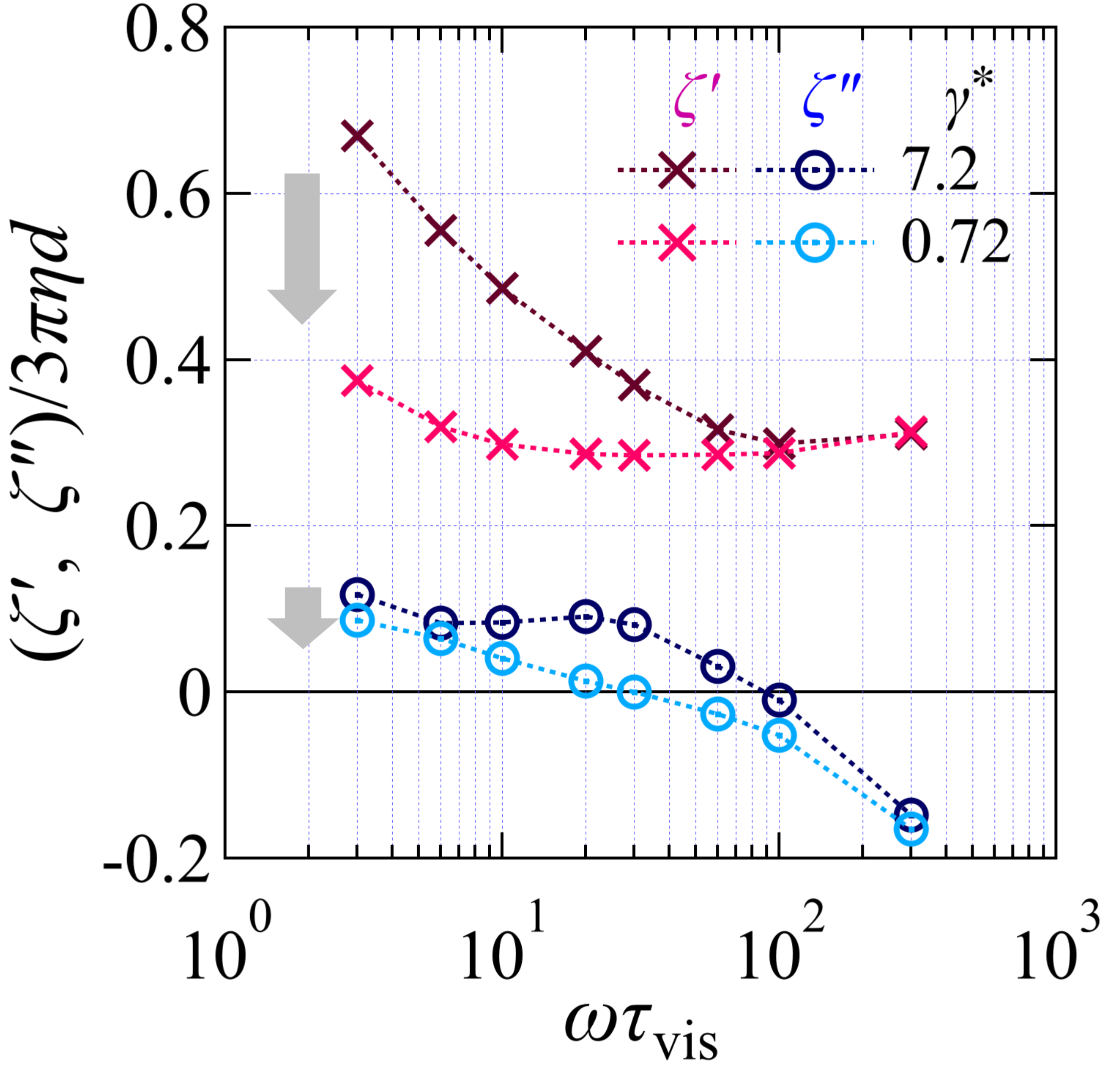
□ Oscillatory motion of a sphere

G. G. Stokes (1851).

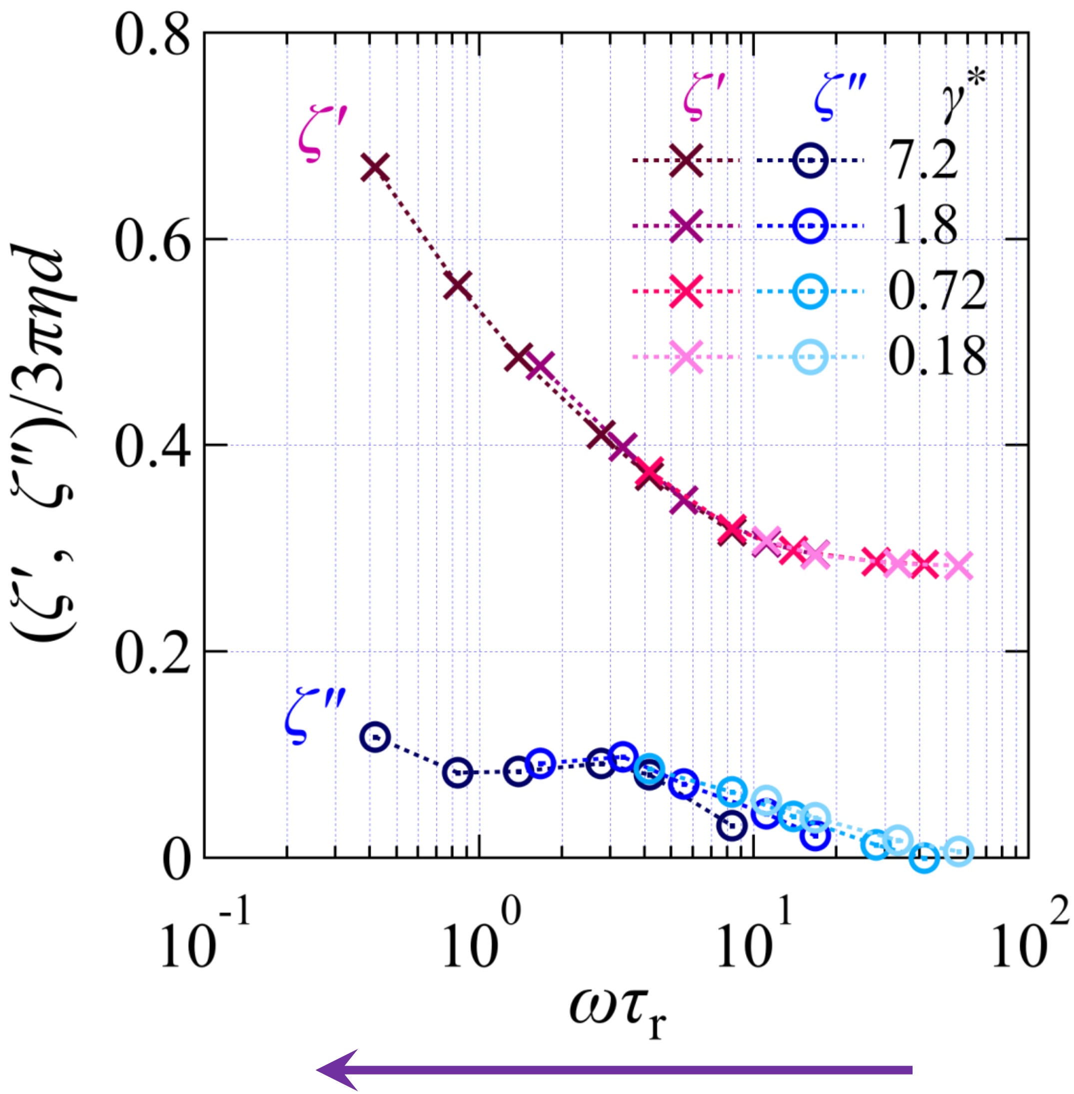


Negative elastic part ← Unsteady flow effects

Effects of Surface Tension



Scaling by Relaxation Time



$$\tau_r = \frac{\eta d}{\gamma}$$ Relaxation time

$$\frac{\eta d \delta_0}{\tau_r} \sim \gamma \delta_0$$

Viscous force ~ Elastic force

$$\omega\tau_r = \frac{\eta d \omega}{\gamma} \propto \frac{\eta U_0}{\gamma}$$

Capillary number

Larger drag coefficients ← Larger surface tension, Lower frequency

Summary

- ◆ Modeling of sticking in mechanics
  - Constraint on relative motions between contacting particles
    → Fixation of contact points
  - Constraint imposed by viscoelastic force

- ◆ Examination of sticking by capillary bridges
  - Calculation of dynamic drag coefficients in shear motion
    → Viscoelasticity of capillary bridges
  - Larger elastic/viscous drag coefficients
    ← Larger surface tension, Lower frequency